3 (Sem–2/CBCS) STA HC2

2022

STATISTICS

(Honours)

Paper : STA–HC–2026

(Algebra)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following ***(any seven)*** as directed : $1 \times 7 = 7$

 (a) A polynomial is said to be complete if all the coefficient are present in the polynomial. *(State True **or** False)*

 (b) If α, β, γ are the roots of the equation $x^3 - 5x^2 - 16x + 80 = 0$, then the product of the roots is

 (i) 5

 (ii) -16

(iii) −80

(iv) None of the above

(Choose the correct option)

(c) State the condition that a matrix A has to satisfy to be an orthogonal matrix.

(d) A matrix A will be an Involuntary matrix if $A^2 =$ .... *(Fill in the blank)*

(e) If two rows or two columns of a determinant be identical, the value of the determinant is

(i) 0

(ii) 1

(iii) None of the above.

(Choose the correct option)

(f) If A be any n-rowed square matrix, then $(Adj\,A)\,A = A\,(Adj) =$.........

(Fill in the blank)

(g) The rank of a unit matrix of order n is

(i) 1

(ii) n

(iii) $n-1$

(iv) None of the above

(Choose the correct option)

(h) Given that for a (5×5) matrix A and $|A| = 59$, find $|3A|$.

(i) Consider the system of homogeneous linear equations

$$(A)_{m\times n}\,(X)_{n\times1} = (O)_{m\times1}$$

and suppose $\rho(A) = r$. Find out the number of linearly independent solutions for this system of equations.

(j) If A and B are two equivalence matrices, then $rank\,(A) = rank(B)$.

*(State True **or** False)*

2. Answer ***any four*** of the following questions : 2×4=8

(a) Solve the equation $x^3 - 3x^2 + 4 = 0$, given that two of its roots being equal.

(b) Examine whether the set

$$S = \left\{ \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}, \begin{pmatrix} 5 \\ 6 \\ 7 \end{pmatrix} \right\} \text{ is}$$

linearly independent or not.

(c) If A and B are symmetric matrices, then show that AB is symmetric iff A and B are commute.

(d) Show that the necessary and sufficient condition for a square matrix A to possess the inverse is that $|A| = 0$.

(e) Show that if two adjacent rows or columns of a determinant are interchanged, the sign of the determinant is changed, whereas its numerical value remaining the same.

(f) Given for a (3×3) matrices $|adj\, A| = 20$, find $|A|$.

(g) Write down the matrix of the following forms and verify that it can be written as matrix products $X'AX$.

$$x^2 - 18x_1x_2 + 5x^2$$

(h) Show that λ is a characteristic root of the matrix A if and only if there exists a non-zero vector X such that $AX = \lambda X$.

3. Answer ***any three*** of the following questions : 5×3=15

(a) If α, β, γ are the roots of the equation $x^3 + qx + r = 0$, find the value of

(i) $(\beta+\gamma)^{-1} + (\gamma+\alpha)^{-1} + (\alpha+\beta)^{-1}$

(ii) $\sum (\beta+\gamma-\alpha)^3$

(b) Show that

(i) Every subspace, S, of V_n has a basis.

(ii) The row rank of a matrix is the same as its rank.

(c) Prove that

$$\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$$

(d) P, Q are non singular matrices. Show that if $A = \begin{bmatrix} P & 0 \\ 0 & Q \end{bmatrix}$, then

$$A^{-1} = \begin{bmatrix} P^{-1} & 0 \\ 0 & Q^{-1} \end{bmatrix}$$

(e) State and prove Cayley-Hamilton theorem.

(f) Show that if A is an idempotent matrix with dimension $n \times n$, then

$$rank(A)+rank(I-n)=n$$

(g) Define positive definite, negative definite and semi-positive definite matrices with examples.

4. Answer the following questions ***(any three)***: 10×3=30

(a) (i) Derive the standard form of a cubic equation. 5

(ii) Solve the equation by Cardon's method

$$x^3-9x-28=0$$ 5

(b) (i) Show that if A, B are two n-rowed square matrices then

$$rank\,(AB) \geq rank(A)+rank(B)-n$$

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(ii) Show that the vectors $X_1=(1, 2, 3)$, $X_2=(2, -2, 0)$ form a linearly independent set. 3

(c) Find the inverse of the matrix

$$S=\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$ and show that the

transform of the matrix

$$A=\begin{bmatrix} b+c & c+a & b-c \\ c-b & c+b & a-b \\ b-c & a-b & a+b \end{bmatrix}$$ by S

is a diagonal matrix.

(d) Show that the equations

$$x+y+z=6$$
$$x+2y+3z=14$$
$$x+4y+7z=30$$

are consistent and solve them.

(e) Show that the every $m \times n$ matrix of rank 'r' can be reduced to the form $\begin{pmatrix} I_r & 0 \\ 0 & 0 \end{pmatrix}$ by a finite chain of E-operations, where I_r is the r-rowed unit matrix.

(f) Show that a necessary and sufficient condition for a real quadratic form $X'AX$ to be positive definite is that the leading principal minors of the matrix A of the form are all positive.

(g) Suppose $X = \begin{bmatrix} 1 & 2 \\ 0 & -1 \\ 3 & 1 \end{bmatrix}$

Evaluate $M = I - X(X'X)^{-1}X'$, where notations have their usual meanings. Show that $M = M^2$ and find the rank of M and M^2.

(h) Determine the characteristic roots and the corresponding characteristic vectors of the matrix

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$
