3 (Sem–3/CBCS) STA HC 3

2021

(Held in 2022)

STATISTICS

(Honours)

Paper : STA–HC–3036

(Mathematical Analysis)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following as directed : 1×7=7

(a) Find the infimum and supremum of the set $\left\{\frac{(-1)^n}{n}; n \in N\right\}$.

(b) Identify the wrong statement :

(i) The set R of real numbers is an open set.

(ii) The set of Q of rationals is an open set.

(iii) The set $\left\{\frac{1}{n} : n \in N\right\}$ is not open.

(c) Show that the series $\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\ldots$ is not convergent.

(d) Give the interpretation of Rolle's theorem.

(e) Suppose Σu_n is a positive term series, such that

$$\lim_{n\to\infty} n\left(\frac{u_n}{u_{n+1}}-1\right)=l.$$

This series converges if

(i) $l>1$

(ii) $l<1$

(iii) $l=1$

(iv) $l=0$

(Choose the correct option)

(f) Which of the following is not correct ?

(i) $\delta = E^{1/2} - E^{-1/2}$

(ii) $\Delta\nabla = \Delta - \nabla$

(iii) $\mu = \frac{1}{2}\left[E^{1/2} + E^{-1/2}\right]$

(iv) $\Delta^2 = E^2 + 2E + 1$

(g) Which of the following is not correct ?

(i) Weddle's rule is more accurate than the Simpson's rule.

(ii) Weddle's rule requires at least seven consecutive values of y.

(iii) In Weddle's rule y is of the form

$$y = ax^6 + bx^5 + cx^4 + dx^3 + ex^2 + fx + g$$

(iv) None of the above

2. Answer the following questions : 2×4=8

(a) Show that the sequence $\{S_n\}$, where

$$S_n = 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n}$$ is not convergent.

(b) If M and N are neighbourhood of a point x, then show that $M \cap N$ is also a neighbourhood of x.

(c) Show that $\sin x$ is uniformly continuous on $[0, \infty]$.

(d) State the properties of divided differences.

3. Answer **any three** of the following questions : 5×3=15

(a) Show that every convergent sequence is bounded and has a unique limit.

(b) Define positive term series. Show that the positive term geometric series $1 + r + r^2 + \ldots$ converges for $r < 1$ and diverges to $+\infty$ for $r \geq 2$.

(c) State and prove first mean value theorem of differential calculus.

(d) (i) Show that

$$\Delta x^m - \frac{1}{2}\Delta^2 x^m + \frac{1.3}{2.4}\Delta^3 x^m - \frac{1.3.5}{2.4.6}\Delta^4 x^m + \ldots m \text{ terms}$$

$$= \left(x + \frac{1}{2}\right)^m - \left(x - \frac{1}{2}\right)^m$$

(ii) Define Limit superior and Limit inferior.

(e) Prove that Newton-Gregory formula is a particular case of Newton's divided formula.

4. (a) (i) If $\lim_{n \to \infty} a_n = l$, then show that

$$\lim_{n \to \infty} \left(\frac{a_1 + a_2 + \ldots + a_n}{n} \right) = l$$ 8

(ii) Verify whether Rolle's theorem is applicable to the function $f(x) = 2 + (x-1)^{2/3}$ in the interval $[0, 2]$ or not. 2

Or

(b) (i) Show that the sequence $\{S_n\}$, where $S_n = \left(1 + \frac{1}{n}\right)^n$ is convergent and that limit $\left(1 + \frac{1}{n}\right)^n$ lies between 2 and 3.

8

(ii) State Cauchy's nth root test. 2

5. (a) (i) State and prove Stirling interpolation formula. 7

(ii) Solve the difference equation

$$y_{k+1} - ay_k = 0,\ a \neq 1$$ 3

Or

(b) (i) Expand $\sin x$ by Maclaurin's infinite series. 8

(ii) State Taylor's theorem with Cauchy's form of remainder. 2

6. (a) (i) State and prove Weddle's rule. 7

(ii) Show that

$$\mu^2 y_x = y_x + \frac{1}{4}\delta^2 y_x$$ 3

Or

(b) (i) Show that

$$\lim_{n\to\infty}\left[\frac{1}{\sqrt{n}} + \frac{1}{\sqrt{n+1}} + \ldots + \frac{1}{\sqrt{2n}}\right] = \infty$$ 4

(ii) Define absolute convergence and conditional convergence.

Show that every absolutely convergent series is convergent. 6
